Mathematical Logic & Structural Proof Theory

Logic Fundamentals, First Order/Higher Order Logic, Natural Deduction, Props as Types, Sequent Calculus

Mathematical logic permeates all of mathematics and programming.

Building on simple propositional logic, a host of richer logics can be constructed to target different needs. For example, First Order Logic is the deductive system used by set theory and is also the basis for Description Logic. Higher Order Logics have richer predicates.

We need to look at what we can represent in logic and what valid claims we can make about such. We also are interested in building our own logics.

One area of logic that is of particular interest is proof theory. Many believe the future of programming will be significantly influenced by automated verification of correctness of code - but what does that mean and how can it be achieved? There are already open source projects pointing the way to much more extensive use of formal verification.

There are a number of sub-fields in proof theory but we focus in on structural proof theory, which has some very interesting features. We are also curious as to what impact treating a proof as a mathematical object has.

Target Audience

This course is aimed at mathematicians and modern developers who need a better grasp of how mathematical logic and proof theory can be used in practice

Prerequisites

Good foundational mathematical education along with some programming experience, as we include exploring logic from a computational viewpoint. Attendees can select which programming language they wish to use, as all concepts will be developed from first principles.

Contents of One-Day Training Course

Logic Fundamentals

Review of logic as branch of mathematics

Propositional Logic

Conjunction, disjunction, negation, conditional, truth tables

Predicate (First Order) Logic

∀ means “for all”
∃ means “there exists”
Quantifiers, more symbols, ..

Higher Order Logic

Predicates themselves having parameters
What variables range over (sets of sets)

Additional Logics

Modal Logic - modal terms and what impact they might have
Temporal Logic – time-based, more ..

Model Theory

Mathematical models
An interpretation gives meaning to symbols in a formal language
When is an interpretation a model
Interpretation function / Domain

Structural Rules

Weakening
Contraction
Exchange
Associativity

Linear Logic

Resources (use once) vs. truth (constant)
Substructural logic
Writing deadlock-free code
Specifications for communication
Session types

Formal Verification

Mathematically proving that code works in all circumstances will always be more desirable than unit testing it for known scenarios

Overview of Proof Theory

What is proof theory?
Many branches – structural proof theory, provability logic, proof mining, automated theorem proving, ...
Structural proof theory includes natural deduction/sequent calculus/hilbert
A proof as a mathematical object- can be manipulated and reasoned about like any other mathematical object

Natural Deduction

Judgment, evidence and witnesses
Depending on what kind of logic that interests us, differing judgments needed
Introduction rule
Elimination rule

Proposition As Types

Relationship to Lambda Calculus
Propositions as types
Proofs as programs
Normalization as evaluation of programs
Nirvana: code is (provable) logic is code

Sequent Calculus

A sequent consists of propositions to the left (ANDed), a turnstile and propositions to the right (OR)
Cut elimination
Importance for linear logic