

Mathematical Logic & Structural Proof Theory

Logic Fundamentals, First Order/Higher Order Logic, Natural Deduction, Props as Types, Sequent Calculus

Mathematical logic permeates all of mathematics and programming.

Building on simple propositional logic, a host of richer logics can be constructed to target different needs. For example, First Order Logic is the deductive system used by set theory and is also the basis for Description Logic. Higher Order Logics have richer predicates.

We need to look at what we can represent in logic and what valid claims we can make about such. We also are interested in building our own logics.

One area of logic that is of particular interest is proof theory. Many believe the future of programming will be significantly influenced by automated verification of correctness of code - but what does that mean and how can it be achieved? There are already [open source projects](#) pointing the way to much more extensive use of formal verification.

There are a number of sub-fields in proof theory but we focus in on structural proof theory, which has some very interesting features. We are also curious as to what impact treating a proof as a mathematical object has.

Contents of One-Day Training Course	
<p>Target Audience This course is aimed at mathematicians and modern developers who need a better grasp of how mathematical logic and proof theory can be used in practice</p> <p>Prerequisites Good foundational mathematical education along with some programming experience, as we include exploring logic from a computational viewpoint. Attendees can select which programming language they wish to use, as all concepts will be developed from first principles.</p>	<p>Logic Fundamentals Review of logic as branch of mathematics</p> <p>Propositional Logic Conjunction, disjunction, negation, conditional, truth tables</p> <p>Predicate (First Order) Logic \forall means “for all” \exists means “there exists” Quantifiers, more symbols, ..</p> <p>Higher Order Logic Predicates themselves having parameters What variables range over (sets of sets)</p> <p>Additional Logics Modal Logic - modal terms and what impact they might have Temporal Logic – time-based, more ..</p> <p>Model Theory Mathematical models An interpretation gives meaning to symbols in a formal language When is an interpretation a model Interpretation function / Domain</p> <p>Structural Rules Weakening Contraction Exchange Associativity</p> <p>Linear Logic Resources (use once) vs. truth (constant) Substructural logic Writing deadlock-free code Specifications for communication Session types</p>
	<p>Formal Verification Mathematically proving that code works in all circumstances will always be more desirable than unit testing it for known scenarios</p> <p>Overview of Proof Theory What is proof theory? Many branches – structural proof theory, provability logic, proof mining, automated theorem proving, ... Structural proof theory includes natural deduction/sequent calculus/hilbert A proof as a mathematical object- can be manipulated and reasoned about like any other mathematical object</p> <p>Natural Deduction Judgment, evidence and witnesses Depending on what kind of logic that interests us, differing judgments needed Introduction rule Elimination rule</p> <p>Proposition As Types Relationship to Lambda Calculus Propositions as types Proofs as programs Normalization as evaluation of programs Nirvana: code is (provable) logic is code</p> <p>Sequent Calculus A sequent consists of propositions to the left (ANDed), a turnstile and propositions to the right (OR) Cut elimination Importance for linear logic</p>